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The Z' reconsidered

A. Ferroglia¹, A. Lorca², and J.J. van der Bij³

*Physikalisches Institut, Fakultät für Mathematik und Physik,
Albert-Ludwigs-Universität Freiburg, D-79104 Freiburg, Germany*

Abstract

We consider the extension of the standard model with an arbitrary number of $U(1)$ gauge fields coupled to baryon-minus-lepton number and/or hypercharge. Under the assumption that A_{fb}^b from the LEP1 experiment is an unlucky fluctuation, we find moderate evidence for the presence of such fields in the precision electroweak data. A relatively large range of the Higgs boson mass is allowed. We discuss the phenomenology of the extra $U(1)$ fields.

1 Introduction

The present day data from the high-energy colliders like LEP and the Tevatron show that almost all data are described by the standard model at the loop level. Therefore the extensions of the standard model (SM) tend to be strongly constrained. Typically such extensions will spoil the agreement with the data through a variety of effects, one of the most important of which is the appearance of flavor changing neutral currents. Even the most popular extension, the minimal supersymmetric extension of the standard model, has to finely tune a number of parameters. This leaves only one type of extensions that are safe, namely extensions with singlet particles. Since the discovery that neutrinos are massive, it is clear that singlet fermions play a role in nature. There are arguments from cosmology, that singlet scalars could be important [1, 2, 3, 4, 5]. In a recent analysis of the Higgs-search data from LEP2 it has been pointed out, that such singlet scalars may already have

¹email: andrea.ferroglia@physik.uni-freiburg.de

²email: alejandro.lorca@physik.uni-freiburg.de

³email: jochum@physik.uni-freiburg.de

been seen as a smeared-out Higgs boson [6]. Given this situation it is natural to ask whether also singlet vector bosons can be present. Therefore we decided to study the most general renormalizable extension of the standard model containing extra gauge bosons, but no extra fermions or scalars. Since the mass of these extra bosons is put in without a Higgs mechanism, these can only be abelian vector bosons. Demanding the absence of anomalies in the gauge currents, the extra vector bosons can only couple to linear combinations of hypercharge (Y) and the difference of baryon and lepton quantum numbers ($B-L$). However they can have an arbitrary mixing with the standard model hypercharge field. Through this mixing, these fields introduce small changes in the couplings of the Z -boson to matter. These couplings are suppressed by a factor $m_Z^2/m_{Z'}^2$. If the masses of the extra bosons are large enough such effects are allowed in the data. Of course if the couplings are infinitesimal, such an extension is always possible. The real question is whether the effects of the new physics can improve the agreement with the data. Even though almost all data are described by the SM [7], there is the somewhat disturbing situation, that the overall fit to the standard model is not very good. The reason is the large difference in $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ from the forward-backward asymmetry A_{fb}^b of the bottom quarks and the measurement from the SLAC SLD experiment. No realistic model has so far been able to explain this difference. It has led some authors [8, 9, 10, 11, 12], to reanalyze the data leaving out A_{fb}^b . In this case one can get a good fit to the data, however with an unphysical Higgs mass of 50 GeV. What we will show in this paper, is that we can get a good fit to the data for a physically allowed Higgs mass, if we allow for one or more extra $U(1)$ fields. We must however emphasize that this works only in the reduced data set without A_{fb}^b . Using the full data set it appears that one has, close to the 95% CL, an unresolvable problem. Within the reduced data set we were able to find confidence level contour plots for the couplings of the extra vector bosons, as a function of the Higgs mass. One can actually allow for a larger range of the Higgs boson mass than in the SM.

The outline of the paper is as follows. In section 2, we describe the models. In section 3 we discuss the Z -boson couplings in relation to the precision experiments. In section 4 we discuss the phenomenology of a single Z' -boson. In section 5 we consider some interesting possibilities with more than one Z' -boson. In section 6 we recapitulate our conclusions.

2 The model

There is a large class of models containing extra neutral vector bosons [13]. We limit ourselves to the simplest, so-called non-exotic extensions [14, 15, 16, 17]. In this type of extension the extra gauge bosons couple universally to the fermion generations and no exotic fermions are present. The model consists of the SM plus an extra number n of $U(1)$ gauge fields. We call the $U(1)$ fields \tilde{C}_μ^i , where $i = 0, 1, \dots, n$. These fields can each have arbitrary couplings to the Y and to B - L . Other couplings are not possible, because these violate the renormalizability of the model through the anomaly. Besides this, we are concerned with the ordinary $SU(2)_L$ gauge fields W_μ^a . The pure gauge field part of the Lagrangian consists of a kinetic and a mass term.

The most general form is:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \sum_{a=1}^3 F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{4} \sum_{i,j=0}^n \tilde{z}_{ij} \tilde{C}_{\mu\nu}^i \tilde{C}_{\mu\nu}^j - \frac{1}{2} \sum_{i,j=0}^n \tilde{m}_{ij}^2 \tilde{C}_\mu^i C_\mu^j, \quad (1)$$

where $\tilde{C}_{\mu\nu}^i$ and $F_{\mu\nu}^i$ are the field strength tensors for the $U(1)$ s and $SU(2)_L$ fields respectively, and the \tilde{z}_{ij} and \tilde{m}_{ij}^2 are the coupling parameters.

Through a linear transformation of fields ($\tilde{C} \rightarrow C$), we can bring the Lagrangian in the canonical form in the diagonal mass basis ($\tilde{z}_{ij} \rightarrow \delta_{ij}$, $\tilde{m}_{ij}^2 \rightarrow \delta_{ij} m_i^2$). In order that the photon stays massless, one of the C_μ fields must be massless; we will call this field $B_\mu \equiv C_\mu^0$. The Lagrangian becomes therefore:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \sum_{a=1}^3 F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} - \frac{1}{4} \sum_{i=1}^n C_{\mu\nu}^i C_{\mu\nu}^i - \frac{1}{2} \sum_{i=1}^n m_i^2 C_\mu^i C_\mu^i. \quad (2)$$

Having the fields in the mass basis, the original couplings of the fields have changed. We have only one extra condition. This is that the B_μ field must not couple to B - L , because otherwise the photon would have a B - L quantum number (Q_{B-L}). The C_μ^i fields have couplings g_Y^i to Y and g_{B-L}^i to B - L .

The coupling to the fermions is then described through the minimal coupling

$$D_\mu \psi = \left(\partial_\mu + i \frac{g}{2} W_\mu^a \tau_L^a + i \frac{g}{2c} \left(s B_\mu + \sum_{i=1}^n g_Y^i C_\mu^i \right) Y + i \sum_{i=1}^n g_{B-L}^i C_\mu^i Q_{B-L} \right) \psi. \quad (3)$$

Here s and c are sine and cosine of the weak mixing angle. The gauge boson coupling to the Higgs field originates from the same covariant derivative, but

the Higgs field has $Q_{B-L} = 0$. After spontaneous symmetry breaking, we therefore find a mass matrix of the form:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \frac{m^2}{c^2} \left(-cW_\mu^3 + sB_\mu + \sum_{i=1}^n g_Y^i C_\mu^i \right)^2 - \frac{1}{2} \sum_{i=1}^n m_i^2 C_\mu^i C_\mu^i. \quad (4)$$

We subsequently assume that $m_i \gg m$ and diagonalize the mass matrix in perturbation theory in the parameters

$$\frac{1}{\mu_i^2} \equiv \frac{m^2}{c^2 m_i^2}. \quad (5)$$

In the following we will ignore effects of $\mathcal{O}(1/\mu_i^4)$.

We recover the SM relation between the photon field (A_μ) and the Lagrangian fields

$$A_\mu = sW_\mu^3 + cB_\mu, \quad (6)$$

while for the Z -boson we find

$$Z_\mu = -cW_\mu^3 + sB_\mu - \sum_{i=1}^n \frac{g_Y^i}{\mu_i^2} C_\mu^i, \quad (7)$$

with the Z -boson mass given by:

$$m_Z^2 = \frac{m^2}{c^2} (1 - a_Y), \quad (8)$$

where we defined

$$a_Y \equiv \sum_{i=1}^n \frac{(g_Y^i)^2}{\mu_i^2}. \quad (9)$$

For the Z' -bosons we find

$$Z_\mu^i = C_\mu^i + \frac{g_Y^i}{\mu_i^2} (-cW_\mu^3 + sB_\mu), \quad (10)$$

with masses

$$m_{Z^i}^2 = m_i^2 \left(1 + \frac{(g_Y^i)^2}{\mu_i^2} \right). \quad (11)$$

With the relations above and $Q = \tau_L^{(3)}/2 + Y/2$ we find for the coupling $i\bar{\psi}\gamma_\mu g_\psi^Z \psi$ of the Z -boson to the fermions:

$$g_\psi^Z = \frac{-g\tau^{(3)}}{4c} (1 - a_Y) \left(1 - 4|Q|(s^2 - c^2 a_Y) + 4\tau_L^{(3)} Q_{B-L} a_{BY} + \gamma_5 \right), \quad (12)$$

where we defined

$$a_{BY} \equiv \frac{c}{g} \sum_{i=1}^n \frac{g_Y^i g_{B-L}^i}{\mu_i^2}. \quad (13)$$

The isospin quantum number $\tau^{(3)}$ is +1 for neutrino and up-quark, -1 for electron and down-quark. For the Z^i couplings to the fermions we find

$$g_\psi^i = \frac{-g\tau^{(3)}}{4c} g_Y^i \left(1 - 4|Q| + \gamma_5 + \frac{1}{\mu_i^2} (1 - 4|Q|s^2 + \gamma_5) \right) + g_{B-L}^i Q_{B-L}. \quad (14)$$

For the Z -pole variables, a_Y and a_{BY} are sufficient to describe the data, but for off-shell quantities also the direct Z' -exchange plays a role. We therefore introduce the quantity

$$a_{B-L} \equiv \frac{c^2}{g^2} \sum_{i=1}^n \frac{(g_{B-L}^i)^2}{\mu_i^2}. \quad (15)$$

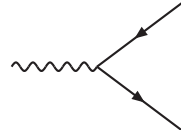
In the case of a single extra $U(1)$ boson one has the relation

$$a_{BY}^2 = a_Y a_{B-L}. \quad (16)$$

When one has more than one extra vector boson this relation is in general not true anymore.

3 Z-boson Couplings and Fits to the Precision Data

In this section we compare the predictions of the model described above with the precision measurements of LEP and SLD at the Z -boson peak. We restrict our analysis to the case of a single Z' -boson. Since $m_{Z'} \gg m_Z$, the Z -peak observables are only sensitive to the modified Z -boson couplings. Effects from direct Z' -boson exchange are suppressed by a factor $\Gamma_Z^2/m_{Z'}^2$. The Feynman rule for the vertex coupling the Z -boson to a fermion f is given by



$$\longrightarrow i\gamma^\mu (V_f + A_f \gamma_5), \quad (17)$$

where the vector and axial couplings V_f and A_f are

$$V_f = -\frac{\tau^{(3)}g}{4c} (1 - a_Y - 4|Q|(s^2 - a_Y) + 4a_{BY}\tau^{(3)}Q_{B-L}), \quad (18)$$

$$A_f = -\frac{\tau^{(3)}g}{4c} (1 - a_Y). \quad (19)$$

In the equations above, $\tau^{(3)} = \pm 1$ is the isospin third component, Q the electric charge and Q_{B-L} is -1 for leptons and $1/3$ for quarks.

In perturbation theory, even at the tree-level, in order to make predictions for physical observables, it is necessary to fix the numerical value of the Lagrangian parameters g , s , and m in terms of the input parameter set α , m_Z and G_F . In the model under study, the fitting equations relating the Lagrangian parameters with the experimental data of the input parameter set are

$$\begin{aligned} 4\pi\alpha &= g^2 s^2, \\ G_F &= \frac{1}{4\sqrt{2}} \frac{g^2}{m^2}, \\ m_Z^2 &= \frac{m^2}{c^2} (1 - a_Y). \end{aligned} \tag{20}$$

By solving the fitting equations, one finds that all the bare Lagrangian parameters depend on a_Y

$$\begin{aligned} g^2 &= g_*^2 + k_g a_Y, \\ m &= m_* + k_m a_Y, \\ s^2 &= s_*^2 - k_s a_Y, \end{aligned} \tag{21}$$

where the constant introduced above have the numerical values

$$\begin{aligned} g_*^2 &= 0.4322, & k_g &= 0.5915, \\ m_* &= 80.94 \text{ GeV}, & k_m &= 55.38 \text{ GeV}, \\ s_*^2 &= 0.2122, & k_s &= \frac{c_*^2 s_*^2}{c_*^2 - s_*^2} = 0.2904. \end{aligned} \tag{22}$$

The theoretical predictions for the physical observables have to be expressed in terms of the quantities g_* , m_* and s_* ($c_*^2 = 1 - s_*^2$). It is important to observe that, applying the Eqs. (22) to the Z -boson coupling in Eq. (12), one has to operate the replacement

$$s^2 - c^2 a_Y \quad \rightarrow \quad s_*^2 - \frac{c_*^4}{c_*^2 - s_*^2} a_Y. \tag{23}$$

In the case in which $Q = -1$, the quantity $1 - 4s_*^2$ is close to zero and therefore sensitive to radiative corrections. This has been well studied within the SM. The main effect comes from the $\gamma - Z$ mixing terms from fermion loops and leads to a shift in the effective value of s_* to $s_* \approx 0.2314$, as measured in the fermion coupling to the Z -boson, which is the numerical value to use in comparison between the SM and the Z' model. The remaining uncertainty

in the prediction is of the order of the radiative corrections combined with Z' effects. Given the experimental errors affecting the measurements of the physical observables, the remaining uncertainties on the theoretical side will not affect the comparison with the data.

Numerically we get the following relations

$$R_l|_{Z'} = R_l|_{\text{SM}}(1 + 0.92a_Y - 1.11a_{BY}) \quad (24)$$

$$\sigma_0^{\text{had}}|_{Z'} = \sigma_0^{\text{had}}|_{\text{SM}}(1 - 0.10a_Y + 2.28a_{BY}) \quad (25)$$

$$\Gamma_Z|_{Z'} = \Gamma_Z|_{\text{SM}}(1 + 0.16a_Y - 1.10a_{BY}) \quad (26)$$

$$\sin^2 \theta_{\text{eff}}^{\text{lept}}|_{Z'} = \sin^2 \theta_{\text{eff}}^{\text{lept}}|_{\text{SM}} - 1.10a_Y - a_{BY} \quad (27)$$

$$m_W|_{Z'} = m_W|_{\text{SM}}(1 + 0.72a_Y) \quad (28)$$

3.1 Analysis of the Z-pole data

In order to test whether there is evidence in the data for this model we used the following precision measurements: the six $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ measurements from LEP1 and SLD, R_l , σ_{had}^0 and Γ_Z also from LEP1. Furthermore the m_W measurements from LEP2 and the Tevatron were used. G_F , α_{em} and m_Z are used as the input parameters of the model. Variation within 1σ of m_t , $\Delta\alpha_h^{(5)}$ and α_s do not affect significantly the following analysis. To see whether a given model, (i.e. a prescribed value of m_H , a_Y and a_{BY}) fits the data, we calculated the predicted values of the above mentioned variables and calculated χ^2 for these values. We varied the parameters over the physical range $m_H > 115$ GeV, $a_Y \geq 0$ and a_{BY} arbitrary.

We proceed as follows. We assume that the world is described by a known value of m_H , a_Y and a_{BY} within the physically allowed range. On the basis of these values a prediction is made for each of the n physical observables x_i . We form the quantity

$$\chi^2 = \sum_{i=1}^n \frac{\left(x_i^{\text{measured}} - x_i^{\text{predicted}}\right)^2}{\sigma_i^2} \quad (29)$$

According to statistical theory this quantity should follow a P_n cumulative probability distribution

$$P_n(\chi^2) = \int_{\chi^2}^{\infty} dy \frac{y^{n/2-1} e^{-y/2}}{\Gamma(n/2)2^{n/2}} \quad (30)$$

We can calculate the probability $P_n(\chi^2)$. This number gives the probability, that a random fluctuation in the data measurements would have a

probability smaller than the actually measured data. If this number is small, it means that the data are not well described by the assumed values of m_H , a_Y and a_{BY} . The 95% confidence level area in the space of m_H , a_Y and a_{BY} consists of the points with $P_n(\chi^2) > 5\%$. It means that the points outside this range are ruled out with a “confidence of 95%”.

In order to see whether any value of the parameters lies within 95% CL area, we looked for the lowest value of χ^2 . The results are shown in Tab. (1). For all models the lowest value of χ^2 was found at $m_H = 115$ GeV, giving $\chi^2/\text{d.o.f.} \simeq 19/11$. This high value indicates a bad fit to the data, as was noticed before in [8, 9, 10, 11, 12]. The presence of the extra $U(1)$ field hardly improves the χ^2 . This is in agreement with the result in [16] for the $a_{BY} = 0$ case. Both the SM and the extended models are barely compatible with the data at the 95% CL.

Table 1: Test of χ^2 for 11 experimental precision data at $m_H = 115$ GeV.

Model	χ^2	$P_{11}(\chi^2)$	m_H	a_Y	a_{BY}
SM	19.1	5.9%	115	–	–
a_Y	19.0	6.1%	115	$3.3 \cdot 10^{-5}$	–
a_Y, a_{BY}	19.0	6.1%	115	0	$4.4 \cdot 10^{-5}$

There are two possibilities that a bad fit can arise. The first is that the model is simply not correct. The other possibility is that the model is correct, but that the data happen to contain an unlucky data point that spoils the fit. This last possibility can be tested for by removing one measurement from the data and seeing whether there is a large change in χ^2 . If we find a very large change in χ^2 by removing one point, we consider this as an indication that this point should not be used in the fit.

We redid the χ^2 analysis removing one of the data points in turn and the results are shown in Tab. (2) for the more significant changes.

Only in the case of removing A_{fb}^b within the context of the extended models are we able to find a model that cannot be excluded within 68% CL. Confidence level contours are given in the Figs. (1–2).

Altogether we consider this analysis as a mild statistical indication, that the A_{fb}^b point is an unlucky fluctuation and that the electroweak data are very well described by the SM with a light Higgs boson and one or more additional Z' -bosons. We notice that the largest part of the improvement comes from the presence of the a_Y term, the additional parameter a_{BY} having a smaller effect. This allows for models of Z' -bosons all coupled to hypercharge only, which is consistent with renormalization.

Table 2: Test of χ^2 for different sets of 10 data points.

Excluded	Model	χ^2	$P_{10}(\chi^2)$	m_H	a_Y	a_{BY}
A_{SLD}	SM	14.8	13.9 %	134 GeV	–	–
A_{SLD}	a_Y	14.8	13.9 %	134 GeV	0	–
A_{SLD}	a_Y, a_{BY}	14.1	16.8 %	115 GeV	0	$-1.7 \cdot 10^{-4}$
σ_{had}^0	SM	16.2	9.5 %	115 GeV	–	–
σ_{had}^0	a_Y	16.1	9.6 %	115 GeV	$3.6 \cdot 10^{-5}$	–
σ_{had}^0	a_Y, a_{BY}	14.0	17.1 %	194 GeV	$8.9 \cdot 10^{-4}$	$-7.4 \cdot 10^{-4}$
A_{fb}^b	SM	13.1	21.9 %	115 GeV	–	–
A_{fb}^b	a_Y	9.8	45.7 %	115 GeV	$3.0 \cdot 10^{-4}$	–
A_{fb}^b	a_Y, a_{BY}	9.5	48.8 %	115 GeV	$1.8 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$

3.2 Comparison with off-shell data

Besides the Z -pole observables and the W -boson mass, there are a number of low energy experiments that are in principle sensitive to the presence of a Z' -boson. Examples are atomic parity violation, the NuTeV experiment, Møller scattering, and the $(g - 2)_\mu$ experiment. We checked the sensitivity of these experiments to the Z' model. None of these experiments has a sensitivity to the a_Y and a_{BY} variables, that is comparable to the sensitivity of the Z -pole observables.

Also the LEP2 experiment is sensitive to the presence of a Z' -boson. A recent analysis of the LEP2 data for lepton production has claimed evidence for the presence of Z' -boson effects [18, 19]. Limits were put on the axial vector and vector couplings of the Z' -boson. Translated into our notation, the following conditions were found:

$$9a_Y + 24a_{BY} + 16a_{B-L} = (65 \pm 30) \cdot 10^{-3}, \quad (31)$$

$$a_Y = (118 \pm 158) \cdot 10^{-3}. \quad (32)$$

We see that the errors in these measurements are very large, so one cannot say that one is testing the model at the same level of precision as with the Z -pole data. The very large value of the couplings implied in the first formula is not very natural compared to the values we derived before. In principle, since we have the new parameter a_{B-L} , one could have an extra contribution coming from a second Z' -boson, coupled relatively strongly to $B-L$, but with little mixing. An alternative explanation could be that one is actually producing directly a spread-out Z' -boson. Such a model is presented

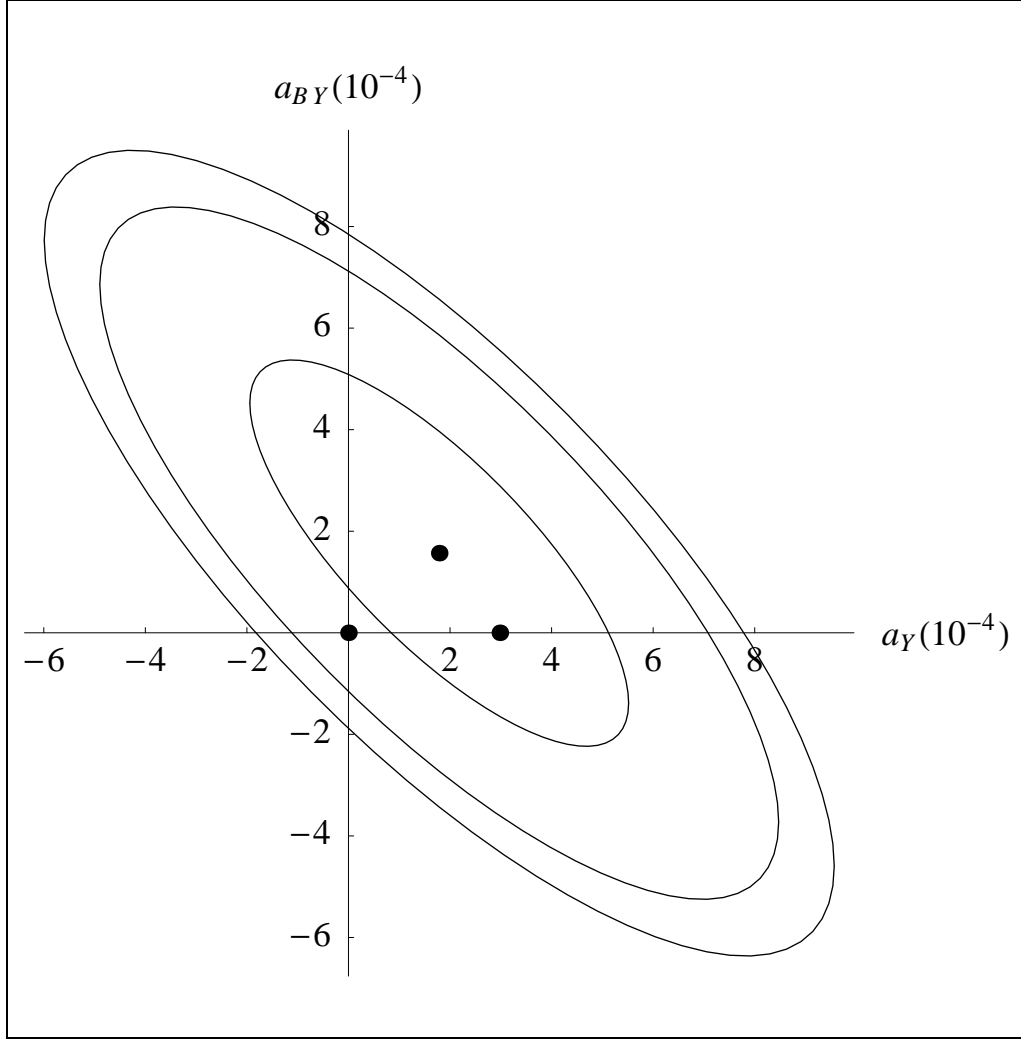


Figure 1: Significance plot for the parameter space (a_Y, a_{BY}) for a fixed $m_H = 115$ GeV. The three contours contain a cumulative probability $P_{10}(\chi^2)$ higher than 32%, 10% and 5% corresponding to the 68%, 90% and 95% CL. The point at the origin describes the SM, while the other two points represent the minimum of the χ^2 distribution for the pure a_Y model and for the a_Y plus a_{BY} model.

in section 5. At the moment one should wait for an independent confirmation of the results in Ref. [18, 19].

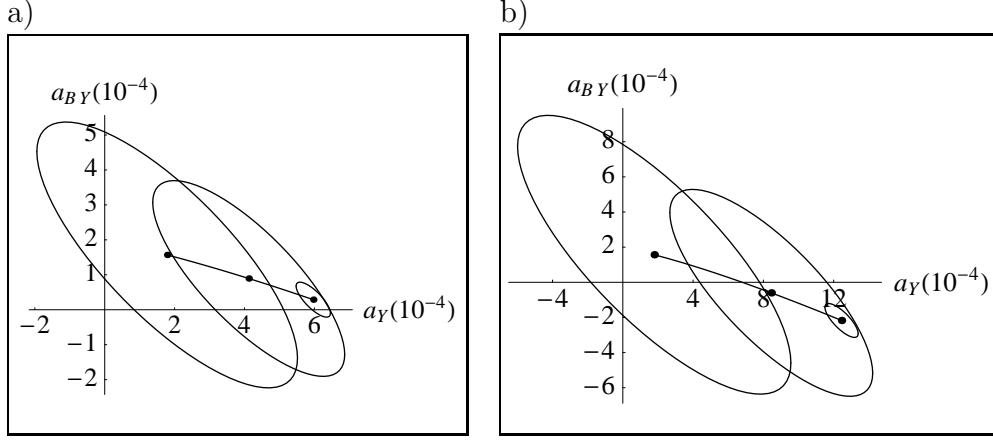


Figure 2: Contours of cumulative probability for $P_{10}(\chi^2)$ distribution for the parameter space $\{a_Y, a_{BY}\}$ at different Higgs masses. a) The different countours describe 68% CL and from left to right masses of 115 GeV, 165 GeV and 215 GeV. The open line moves along the set of minima for the χ^2 up to $m_H = 217$ GeV. b) The countours describe 95% CL and from left to right masses of 115 GeV, 300 GeV and 495 GeV. The open line moves along the set of minima for the χ^2 up to $m_H = 500$ GeV

4 Phenomenology of Z'

4.1 Renormalization group analysis

In order to study the phenomenology of the Z' -boson one needs its mass, production cross sections, and decay branching ratios. These can be computed when coupling constants and mass are known. From the precision data we can put limits on the relative values of the B - L and Y couplings. But since the limits from the precision measurements are in the generic form $g^2/m_{Z'}^2$, we have no direct information on the mass itself. For low masses one can find limits from direct searches. At first sight there appears to be no way to give an upper limit to the mass of the Z' -boson. A very heavy boson might have a large coupling giving similar effects to a light boson with small couplings. Therefore, to get an upper limit, we need more theoretical input. Indeed it is the case that one cannot arbitrarily enlarge the couplings. Since the vector bosons are abelian, their effective couplings grow via renormalization group and will become infinite at the Landau pole. If one demands that the Landau pole is much larger than the electroweak scale, one can get an upper limit to the mass of the Z' -boson. We will give somewhat qualitative limits using the one-loop running of the couplings constants due to the fermions

and demand that the Landau pole lies beyond the Planck mass scale. In this case the new $U(1)$ fields would not be too different from the SM gauge fields. The analysis is slightly complicated, due to the fact that the renormalization group mixes the different fields [20, 21, 22]. We first define the following auxiliary variables

$$t \equiv \frac{\log(\frac{Q^2}{m_Z^2})}{24\pi^2}, \quad g_1 \equiv g \frac{s}{c}, \quad g_2 \equiv g \frac{g_Y^1}{c}. \quad (33)$$

The renormalization group equations become

$$\begin{aligned} \frac{dg_1}{dt} &= 5g_1^3, \\ \frac{dg_{B-L}}{dt} &= g_{B-L} (8g_{B-L}^2 + 5g_2^2 + 8g_2g_{B-L}), \\ \frac{dg_2}{dt} &= g_1^2 (8g_{B-L} + 10g_2) + g_2(8g_{B-L}^2 + 5g_2^2 + 8g_2g_{B-L}). \end{aligned} \quad (34)$$

Their solution is given algebraically by using the derived equations

$$\begin{aligned} \frac{5g_2 + 4g_{B-L}}{g_{B-L}g_1^2} &= \text{constant}, \\ \frac{5g_2^2 + 5g_1^2 - 8g_{B-L}^2}{g_{B-L}^2g_1^2} &= \text{constant}, \\ \frac{1}{g_1^2} + 10t &= \text{constant}. \end{aligned} \quad (35)$$

By demanding that there is no Landau pole before the Planck mass we find the allowed region in the $g_2(0)$ versus $g_{B-L}(0)$ plane given in Fig. (3).

Unfortunately, the constraints we get from this condition are not very strong. For example, if we take the $a_{BY} = 0$ case, one finds the limit $g_2(0) < 0.42$. This corresponds to a limit

$$m_{Z'}^2 < 0.32 \frac{m_Z^2}{a_Y}. \quad (36)$$

So, even for the relatively large value of $a_Y = 4 \cdot 10^{-4}$, one finds a limit of $m_{Z'} < 2.6$ TeV. For smaller values of a_Y the limit gets correspondingly higher. Therefore, even though it is plausible that the Z' -boson is in the low TeV region and thereby within reach of the LHC, this is not quite guaranteed. Stronger upper limits on the Z' -boson mass can only be derived, when more theoretical information is put in, for instance by demanding some unification of coupling constants. Assuming Eq. (31) to be true we can get better limits,

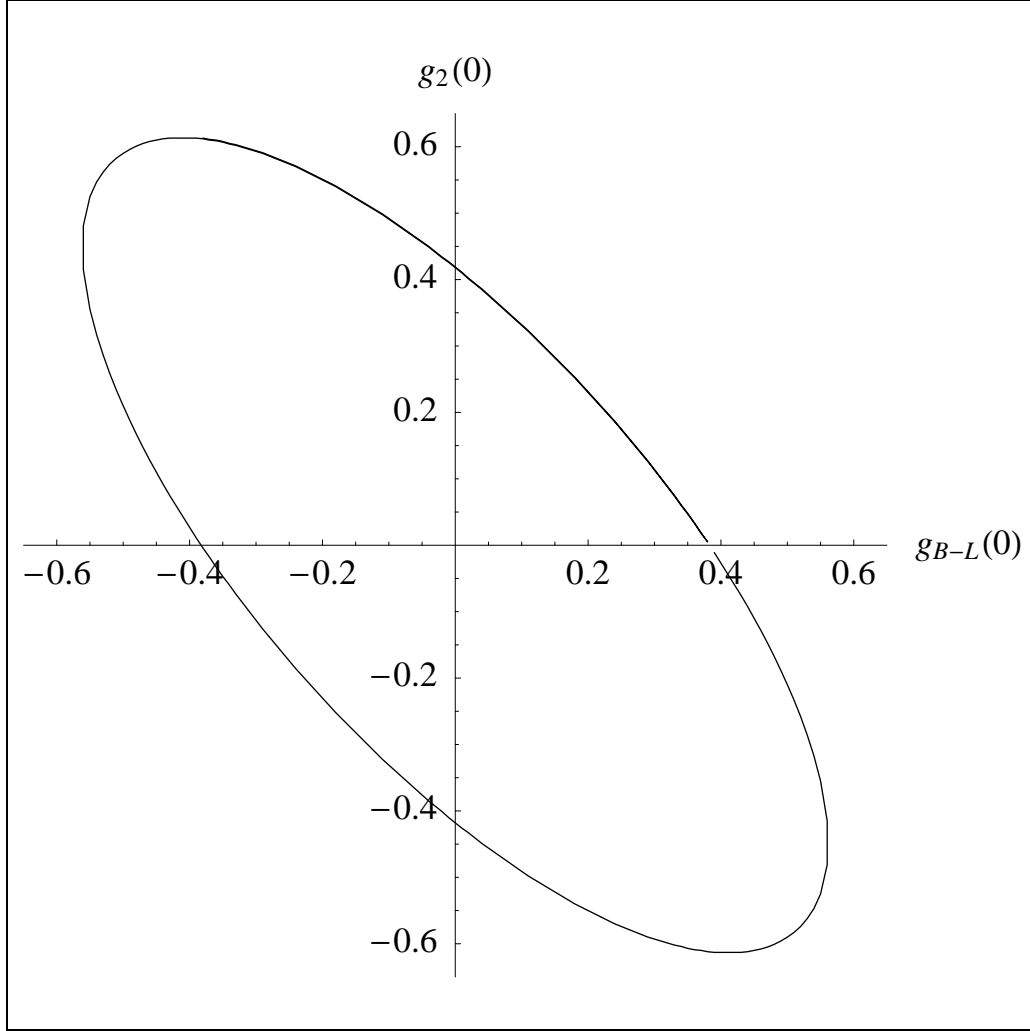


Figure 3: Region of allowed coupling strength at the weak scale.

due to the effect that in this case the a_{B-L} term is the dominant one. Using similar reasoning as above one finds

$$m_{Z'} < 800^{+290}_{-140} \text{ GeV}. \quad (37)$$

In this case the Z' -boson could well be within reach of the Tevatron at high luminosity.

4.2 Collider limits

Nowadays the best limits on the presence of a Z' -boson come from the Tevatron collider. The principle guiding the search is straightforward [23]. One

uses the decay of the Z' -boson into an electron-positron pair and looks for a peak in the invariant mass. In addition one can use information on the forward-backward asymmetry of the leptons. One therefore needs a prediction for the production cross section and for the branching ratio into leptons. Moreover, when one is working within a specific model, one can use the distribution of the leptons in the center of mass angle $\cos\theta^*$, leading to somewhat different limits for different models. This search has recently been made at the Tevatron [23] for the class of models discussed in Ref. [17], but not for the specific models presented here.

In the following we use the narrow width approximation and ignore interference with Z -boson and photon exchange. The production cross section times branching ratio (BR) is well described by the formula⁴:

$$\sigma(p\bar{p} \rightarrow Z' \rightarrow e^+e^-) = \frac{\pi}{12s} \text{BR}(Z' \rightarrow e^+e^-) \times (c_u w_u(s, M_{Z'}^2) + c_d w_d(s, M_{Z'}^2)), \quad (38)$$

where

$$c_u = V_u'^2 + A_u'^2, \quad (39)$$

and

$$c_d = V_d'^2 + A_d'^2. \quad (40)$$

In the above formulas the V' and A' are the vector and axial-vector couplings of the quarks to the Z' as given in Eq. (14). Furthermore w_u and w_d are the luminosities for up-type quarks and down-type quarks, s the total energy squared of the collision in the center of mass system.

The predicted branching ratio to electron-positron is given in our model approximately by

$$\text{BR}(Z' \rightarrow e^+e^-) \simeq \frac{\Gamma(Z' \rightarrow e^+e^-)}{\sum_f \Gamma(Z' \rightarrow f\bar{f})}, \quad (41)$$

where we have neglected the effect of other than fermionic-pair decays. For generic massive fermionic-pair decay of a single Z' we have

$$\Gamma(Z' \rightarrow f\bar{f}) = \frac{m_{Z'}}{12\pi} \sqrt{1 - \frac{4m_f^2}{m_{Z'}^2}} \times \left[\left(1 - \frac{m_f^2}{m_{Z'}^2}\right) (V_f'^2 + A_f'^2) + 3 \frac{m_f^2}{m_{Z'}^2} (V_f'^2 - A_f'^2) \right]. \quad (42)$$

⁴c.f. with Eqs. (3.8-9) from [17]

At the parton level one can give simple formulas for the forward-backward asymmetry in the center of mass system.

$$A_{\text{fb}}^{e,q} = 3 \frac{1}{R_e^{-1} + R_e} \frac{1}{R_q^{-1} + R_q}, \quad (43)$$

where the function $R_f \equiv V'_f/A'_f$ has the following values

$$R_f = \begin{cases} -3 - 4 \frac{a_{BY}}{a_Y}, & \text{for } f = e, \\ -\frac{5}{3} - \frac{4}{3} \frac{a_{BY}}{a_Y}, & \text{for } f = u, \\ -\frac{1}{3} + \frac{4}{3} \frac{a_{BY}}{a_Y}, & \text{for } f = d. \end{cases} \quad (44)$$

For $\bar{p}p$ collisions the measured asymmetry is weighted with the quark luminosities. In pp collisions the overall asymmetry disappears and one must resort to rapidity dependent asymmetries.

Now we address the question, whether the limits derived in section 3 can be used to make general constraints on the expectations at the Tevatron. Actually the range of parameters as implied in Fig. (2) is too large to give much of a prediction even at a 68% CL. Therefore to get an impression of a possible explicit phenomenology we present results with additional constraints. Actually the analysis of the precision data has shown, that the biggest improvement comes from the introduction of the a_Y parameter, the additional a_{BY} parameter giving a smaller improvement to the fit to the data. It is therefore reasonable to consider the phenomenology, assuming g_{B-L} to be absent. This is a consistent condition under renormalization and forms an interesting class of models by itself.

Since the properties of the Z' -boson are strongly correlated with the mass of the Higgs boson, we are interested in knowing what the most likely expectation tells us for different Higgs masses. Therefore we also consider the set of models which move along the χ^2 minimization-line with increasing m_H .

In principle the model dependence is largely contained in the parameters c_u and c_d , defined above. As argued in Ref. [17] it would be useful to have lower limits on the Higgs-mass presented in the $c_u - c_d$ plane. Unfortunately such a comparison has not been presented in the literature. Instead we proceeded estimating the lower limits for the allowed mass of the Z' -boson as a function of either a_Y or m_H , depending on the restrictions mentioned above. We calculated these ranges as follows: we used a LO program to predict the total cross section as a function of the coupling constant and the mass. Then, this cross section was normalized to the sequential Z' -boson, which has a lower limit of 850 GeV [23]. We then connected the lower bounds of $m_{Z'}$ within the two studied models. The derived bounds are of course somewhat

qualitative, because the sensitivity to the $\cos\theta^*$ distribution is not exactly modeled this way. A precise analysis would require taking into account a bidimensional distribution, including the angular one as well, whereby one cannot ignore detector effects [23]. However this needs a detailed simulation of the detector and comparison with the actual data, which is beyond the scope of this paper. To derive the upper limits we used the results from the previous section.

In the analysis of the one-parameter model ($a_{BY} = 0$), we limit ourselves to the 68% confidence interval

$$0.8 \cdot 10^{-4} < a_Y < 6.5 \cdot 10^{-4}, \quad (45)$$

which we used to scan the possible lower and upper limits for the Z' -boson as shown in Fig. (4).

As an alternative we considered the best-fitted models for different values of m_H . In these models the parametrized equations used were

$$a_Y(m_H) = \left(-114 - 7 \log\left(\frac{m_H}{\text{GeV}}\right) + 7.3 \log^2\left(\frac{m_H}{\text{GeV}}\right) \right) \cdot 10^{-5}, \quad (46)$$

$$a_{BY}(m_H) = \left(-42.2 + 41 \log\left(\frac{m_H}{\text{GeV}}\right) - 6.1 \log^2\left(\frac{m_H}{\text{GeV}}\right) \right) \cdot 10^{-5}, \quad (47)$$

being accurate in the interval

$$115 \text{ GeV} \leq m_H \leq 500 \text{ GeV}. \quad (48)$$

Here in order to extract the maxima of the $g_2(0)$ coupling domain, we projected the best estimates of a_Y and a_{BY} onto the ellipse of Fig. (3), obtaining thus a non-linear mapping $m_H \mapsto g_Y^1$, which finally can be transformed, similarly as in Eq. (36), to a $m_{Z'}$ upper limit.

The result for this analysis is shown on Fig. (5). In order to avoid a wrong interpretation, we point out that the limits shown in this picture do not correspond to equal values of confidence level at each m_H . They just provide the range for the best fit. Actually the confidence limit from the precision data fit ranges between 51% at $m_H = 115 \text{ GeV}$ and 95% at $m_H = 500 \text{ GeV}$.

Even though the approach is somewhat limited, it suffices to retrieve three main features. First, the Tevatron limits a large part of the a priori allowed masses. Second, there is a range that could still lead to discovery of a Z' -boson at the Tevatron with more integrated luminosity. Third, we have not enough restrictions on the upper limit to guarantee discovery at the LHC. The reason is that a_Y can be quite small. However if the Higgs mass is larger than allowed within the standard model fits, then a_Y becomes larger and one should be able to find a Z' -boson at the LHC. For the Z' search at the LHC, the models we discussed are similar to other models and present no particular difficulties, therefore we refer to Refs. [24, 25].

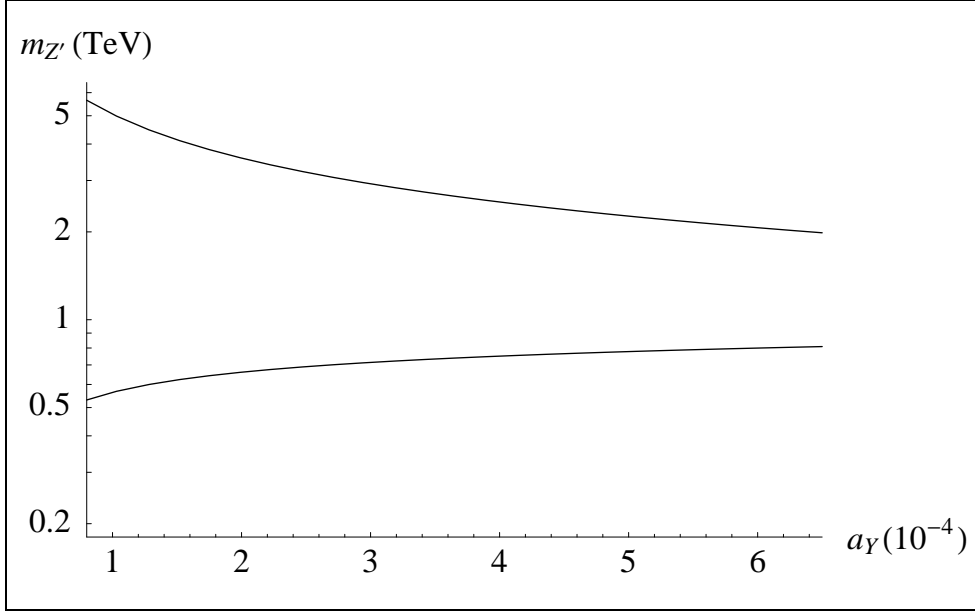


Figure 4: Lower and upper bounds on the Z' -mass for models with $g_{B-L} = 0$.

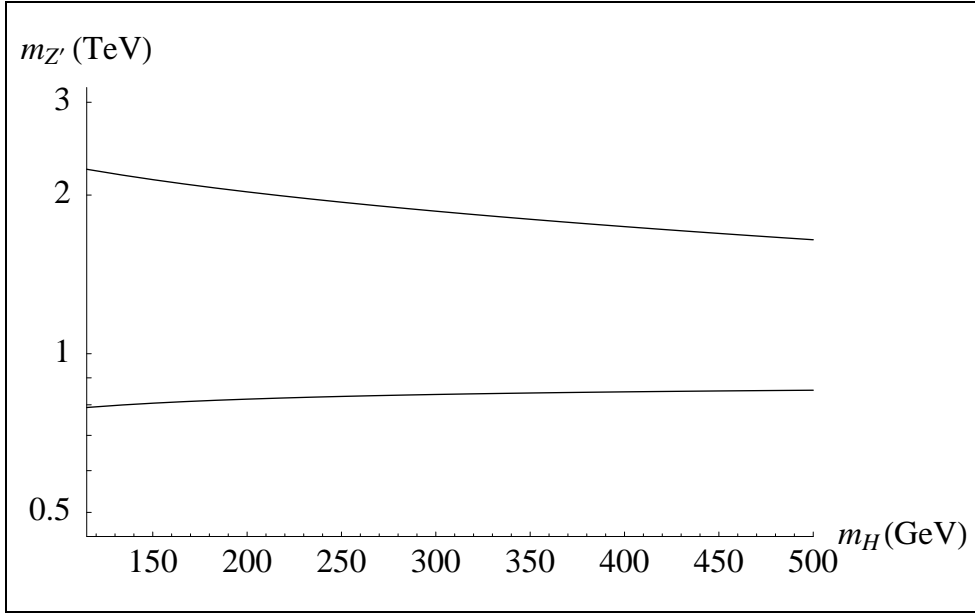


Figure 5: Lower and upper bounds on the Z' -mass for the best model values (a_Y, a_{BY}) as a function of m_H .

5 Multiple Z' -bosons

The formalism as presented in section 2 leaves open the possibility of having more than one extra vector boson. From the precision data there is no possibility to get information about the actual number of Z' -bosons, because one measures only the parameters a_Y and a_{BY} , that are formed by sums over all existing vector bosons. If one adds only a finite number of vector bosons there will be more Z' -bosons with different sets of couplings and the renormalization group running becomes more complicated. However no essentially new aspects appear. The situation becomes more interesting when one allows for an infinite number of extra fields. In particular one can also allow for vector fields moving in more than four dimensions. However for general couplings this will lead to a non-renormalizable theory, since higher dimensional gauge fields coupled to fermions form operators of a dimension higher than four. Here the comparison with the data from section 3 gives a useful hint. It was found that the major improvement in comparison with the data comes from the introduction of the a_Y term. The additional a_{BY} term gives a smaller improvement in χ^2 . It is therefore natural to consider a model where all $U(1)$ fields couple to hypercharge only. In the mass-basis, that we used so far, we then have a large number of fields all coupled to hypercharge with a certain strength. In this case however it maybe advantageous to study the model in the hypercharge basis. One then has one field coupled to hypercharge and further fields coupled to nothing. These extra fields make their presence known only through the mass mixing with the hypercharge field. Since a mass term for a vector boson has a mass dimension $d - 2$ as an operator, one can allow for such vector fields as long as $d \leq 6$. As an example we therefore take an abelian sector with a 4-dimensional field B_μ , coupled to hypercharge and a d -dimensional field A_μ coupled to nothing. We allow for a 4-dimensional mass term, a mixing mass term and a d -dimensional mass term.

The Lagrangian becomes

$$\begin{aligned}
 \mathcal{L}_{\text{gauge}} = & - \int d^4x \left(\frac{1}{4} B_{\mu\nu} B_{\mu\nu} + \frac{1}{2} M_4^2 B_\mu B_\mu \right) \\
 & - \int d^d x \left(\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} M_5^2 A_\mu A_\mu \right) \\
 & - \int d^d x \prod_{i=1}^{d-4} \delta(x_{4+i}) M_{\text{mix}}^{4-\frac{d}{2}} A_\mu B_\mu.
 \end{aligned} \tag{49}$$

We consider the case that the A_μ fields move in a flat open space. By first compactifying the higher dimensions and subsequently taking the continuum

limit one can derive a hypercharge-boson propagator with a nontrivial Källén-Lehmann spectral density. The analysis follows the treatment of the Higgs-singlet mixing in Ref. [6]. For a detailed derivation we refer to this paper. Here we only give the result.

The hypercharge propagator becomes of the form

$$D_{\mu\nu}^{BB}(q^2) = \delta_{\mu\nu} \left[q^2 + M^2 - \mu_{\text{lh}}^{8-d} (q^2 + m^2)^{\frac{d-6}{2}} \right]^{-1}. \quad (50)$$

The masses M , m and μ_{lh} are the free parameters of the model. The scale μ_{lh} stands for low-to-high-dimensional mixing mass and measures the mixing of the high($=d$) dimensional vector and the low($=4$) dimensional vector. The propagator contains a particle peak and a continuum. In order to guarantee that the particle peak is at mass zero, which is necessary to have a massless photon in the theory, we must take:

$$M^2 m^{6-d} = \mu_{\text{lh}}^{8-d}. \quad (51)$$

We now consider the simple cases of integer dimensions 4, 5 and 6. In the four dimensional case we should recover the original model. This is indeed what happens. One finds a single Z' -boson, with

$$m_{Z'}^2 = M^2 + m^2, \quad (52)$$

and

$$a_Y = \sin^2 \theta_W \frac{M^2}{m^2 (M^2 + m^2)}. \quad (53)$$

Next we consider the case $d = 5$. The propagator becomes of the form:

$$D_{\mu\nu}^{BB}(q^2) = \delta_{\mu\nu} \left[q^2 + M^2 - mM^2 (q^2 + m^2)^{-\frac{1}{2}} \right]^{-1}. \quad (54)$$

This corresponds to a Källén-Lehmann spectral density:

$$\rho(s) = \frac{2m^2}{2m^2 + M^2} \delta(s) + \frac{\theta(s - m^2)}{\pi} \frac{mM^2 (s - m^2)^{\frac{1}{2}}}{(s - m^2)(s - M^2)^2 + m^2 M^4}. \quad (55)$$

We therefore find one massless particle and a massive continuum. After spontaneous symmetry breaking the massless excitation becomes the photon. The massive fields form a spread-out hypercharge-coupled Z' -boson. As a consequence we find:

$$a_Y = \sin^2 \theta_W \int_{m^2}^{\infty} \frac{ds}{2\pi m s} \frac{(2m^2 + M^2)M^2 (s - m^2)^{\frac{1}{2}}}{(s - m^2)(s - M^2)^2 + m^2 M^4}. \quad (56)$$

Finally we consider the case $d = 6$. This case is special, since it corresponds to the limiting dimension, where the theory is still renormalizable. Using a limiting procedure around $d = 6$ the propagator can be written as:

$$D_{\mu\nu}^{BB}(q^2) = \delta_{\mu\nu} \left[q^2 + M^2 + \mu_{\text{lh}}^2 \log \left(\frac{q^2 + m^2}{\mu_{\text{lh}}^2} \right) \right]^{-1}. \quad (57)$$

The spectrum has a massless pole when

$$M^2 + \mu_{\text{lh}}^2 \log \left(\frac{m^2}{\mu_{\text{lh}}^2} \right) = 0. \quad (58)$$

The corresponding Källén-Lehmann spectral density is:

$$\rho(s) = \frac{m^2}{m^2 + \mu_{\text{lh}}^2} \delta(s) + \theta(s - m^2) \frac{\mu_{\text{lh}}^2}{\left[s - \mu_{\text{lh}}^2 \log \left(\frac{s - m^2}{m^2} \right) \right]^2 + \pi^2 \mu_{\text{lh}}^4}. \quad (59)$$

Correspondingly one has:

$$a_Y = \sin^2 \theta_W \int_{m^2}^{\infty} ds \frac{m^2 + \mu_{\text{lh}}^2}{m^2 s} \frac{\mu_{\text{lh}}^2}{\left[s - \mu_{\text{lh}}^2 \log \left(\frac{s - m^2}{m^2} \right) \right]^2 + \pi^2 \mu_{\text{lh}}^4}. \quad (60)$$

Since such a spread-out Z' -boson has no direct mass peak, its detection could be quite difficult at hadron colliders. Only an analysis together with an experimental detector simulation can give a reliable answer about the limitations in detection here. An important role could be played by the correlations for the outgoing leptons. At the Tevatron such a correlation can be measured directly, but at the LHC a rapidity dependent analysis will be needed. It seems even possible, that such a Z' could have been produced at LEP2, but has been overlooked because of its rather non-specific signature. A re-analysis of the data at LEP2 with this possibility in mind might be useful.

6 Conclusion

We studied the class of models of multiple $U(1)$ fields coupled only to linear combinations of hypercharge and baryon-minus-lepton number, the so-called non-exotic $U(1)$ fields. We took a careful look at the Z -pole precision data and argued the case, agreeing with some considerations in the literature, that one could reasonably analyze the data without the A_{fb}^b point. In the subsequent analysis we found a moderate indication for the existence of extra $U(1)$ fields. We studied the phenomenology of the extra Z' -bosons, concluding that it is possible that the Tevatron could find the Z' -boson and likely,

but not entirely certain, that the LHC will find it. Furthermore we pointed out the interesting, but somewhat disturbing possibility, that the $U(1)$ fields come from higher dimensions, leading to a spread-out Z' -boson, whose study might be severely limited at a hadron collider. If the energy would be high enough to produce it, a electron-positron collider could study such a spread-out signal without severe difficulties.

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